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Probabilistic approach in modelling dynamic fracture problems

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Abstract

The paper represents a probabilistic approach to the finite element modelling of dynamic fracture problems. It is proposed to model internal structural defects and inhomogeneities using the spatial distribution of strength characteristics according to the normal distribution law. The probabilistic approach to modelling dynamic fracture is provided by the introduction of one additional parameter – the dispersion in the distribution of strength material properties. This approach provides the probabilistic nature to the initiation and development of cracks in the material at any scale level: macro, meso and micro level. There are no restrictions on the size of computational mesh and elements, the fracture criterion and the material models to apply this approach. The probabilistic approach is also applicable to multilevel modelling using the appropriate distribution of inhomogeneities. This approach does not require a detailed study of the material structure, which enhances the predictive nature of computations. The numerical results of exploding cylinder tests and the penetration of thin targets by a projectile are presented. Numerical results are in good qualitative and quantitative agreement with experimental data.

Keywords

Probabilistic approach, dynamic fracture, numerical simulation

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Introduction

The introduction of a probabilistic factor in fracture mechanics is an urgent problem both for constructions and for local fracture. Macroanalysis, as a rule, uses statistical-probabilistic models that take into account the random nature of load, geometric parameters or the presence of internal defects.[1–4] Numerical simulation of a real material structure during deformation and the fracture of solids is a more difficult problem. At present, a multilevel approach which considers the structure of the material at micro- and mesoscale levels is actively developed.[5–7] However, this approach is mainly used to determine the effective (averaged) characteristics of materials in the area comparable to the representative volume. Theoretical estimates of material characteristics[8] also do not take into account the probabilistic nature of the distribution of inhomogeneities in the structure of the material. At the same time, studies show that the distribution of inhomogeneities in the materials structure is stochastic.[9]

In many cases, numerical simulation of the dynamic deformation and fracture of solids has to reliably predict the fracture behaviour, the size and shape of the fragments formed. The process of fracture and fragmentation of deformable bodies is probabilistic, which is determined by the stochastic distribution of defects and inhomogeneities in the structure of the material.[9,10] The most obvious probabilistic nature of fragmentation was demonstrated experimentally and numerically for the fracture of cylindrical shells under explosive loading.[11–13] The initial loading conditions in this case have axial symmetry; however, during loading, deformations are concentrated in the inhomogeneities of the material structure. Growing cracks form stress relaxation zones, which determines the characteristic size of the fragments. Due to the stochastic distribution of defects and inhomogeneities in the structure of the material, fragmentation acquires a probabilistic nature and a substantially asymmetric spatial form. The stress relaxation processes in the inhomogeneities of the material structure strongly affect the fracture

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behaviour during the interaction of projectiles with targets, which can lead to the appearance of radial cracks (for brittle materials) or the fracture in the form of 'petals' for the plate target.

A multilevel approach in numerical simulation considers the real material structure, which, nevertheless, cannot be a solution to the probabilistic fracture behaviour. This approach cannot be used for the simulation of the fracture of real objects, since it requires serious studies of the internal structure and the development of a rather complicated model for numerical simulation, as well as the use of a very fine computation mesh. Such conditions impose significantly higher requirements on computing resources, significantly extend the computation time and decrease the opportunities for parametric studies. However, a crucial point in a multilevel approach is that information on the real internal structure of the material does not allow the probabilistic fracture behaviour to be simulated. The problem is that the distribution of inhomogeneities is stochastic at all levels (micro, meso, macro).

For a long time, the probabilistic approach to solving problems of fracture and fragmentation has been limited by the analytical dependences of the fragment spectrum parameters on the strain rate. In this case, the Mott model and its modifications Grady, Gilvarry and others were used.[9] Most researchers used the Mott approach to simulate fracture processes.[11,14–16] This approach is based on a simplified one-dimensional model, which does not guarantee the reliability of determining the spectrum of fragments or the velocity field of real fragments. At present, the development of computer technologies allows a promising approach to be used for the numerical simulation of fracture to take into account the inhomogeneities of the internal structure of solid by distributing the strength characteristics.[11–13,15,16] The importance of this approach is confirmed by the fact that the distribution of structural inhomogeneities is included in widely used numerical codes for fracture computations.[16]

This paper presents a probabilistic approach to the simulation of dynamic fracture problems. Internal structural defects and inhomogeneities are simulated using the spatial distribution of strength characteristics according to the law of normal distribution. The probabilistic approach to the simulation of dynamic fracture is provided by the introduction of another parameter (dispersion) in the distribution of the strength properties of the material. The proposed approach is applicable to all materials and does not require special experiments for a preliminary study of their structure.

A probabilistic approach to simulation of structural inhomogeneities

The classical approach to the simulation of deformation and fracture does not include stochastic distribution of material inhomogeneities, which provide relaxation of local stresses and the formation of microcracks at the initial stages of the loading process. To provide the probabilistic deformation and fracture behaviour, which is close to reality, strength characteristics should be distributed over the volume of the sample with a specified dispersion of the limiting material states. With this approach, information on the real internal structure of the material is not mandatory. It becomes possible to use the effective values of the physicomechanical and strength characteristics for numerical simulation using the probabilistic approach.

For numerical simulation of fragmentation problems, regardless of whether the mesh or meshless method is used (SPH method[17]), the material is represented by a model with discrete parameters relative to computational nodes and cells.[10] To simulate an inhomogeneous structure, the initial distribution of strength material characteristics should be changed in the proposed probabilistic approach. This paper proposes to modify the initial distribution of yield strength and fracture criterion. In the material models used, as a rule, these parameters are independent, since they depend on various structural defects and their distributions can also be considered independent.

At present, the two-parameter Weibull distribution (1), which is obtained from the power 'risk function' Mott, is usually used to set the initial inhomogeneities in numerical simulation. However, many studies mark the insufficient validity of the Weibull distribution, which leads to the need to select model parameters for better agreement of numerical results with experiments. The distribution of inhomogeneities in the structure of the material is influenced by a large number of independent parameters. In the probability theory, this directly corresponds to the law of normal distribution. This paper, in contrast to the works[10,13–15] that use the Weibull distribution, proposes to use the normal (Gaussian) distribution as theoretically justified and more convenient for simulation.

$$\frac{dp}{dx} = \frac{\lambda}{m_0}\left(\frac{x}{m_0}\right)^{\lambda-1} e^{-\left(\frac{x}{m_0}\right)^{\lambda}} \tag{1}$$

The Weibull distribution used to describe the distribution of ultimate values are usually very close to the normal law; however, using two parameters that do not have a physical meaning gives some uncertainty and can lead to potential errors. The Weibull distribution is asymmetric, therefore setting the physically sensible distribution boundaries and calculating the median and maximum are also a serious difficulty. In addition, it is worth noting that generators generating a random variable obeying the Weibull distribution are in the lack and, perhaps, this keeps many researchers from using the probabilistic approach in the dynamic mechanics of a deformable solid. The normal distribution that uses widespread random number generators for any programming language is deprived of all these disadvantages and greatly simplifies the task.

Setting initial inhomogeneities

Characteristics such as density, shear modulus, bulk compression modulus for a gradientless single-phase material are practically independent of the number of defects and can be considered to be constant when distributed over the volume. At the same time, parameters such as yield strength, tensile strength, maximum strains, and other constants that determine the beginning of fracture in various strength theories and the fracture criteria directly depend on the number and size of defects and should be randomly distributed over the volume with the dispersion depending on the degree of homogeneity of the material.

To simulate initial inhomogeneities, the approach is used as follows. A certain parameter or parameters, as a rule, the ultimate value of fracture, chosen as a fracture criterion or yield strength, is distributed in cells or nodes (depending on the chosen approach to the fracture description) of the computational domain according to the normal distribution law. The distribution is performed using a modified random number generator which produces a value that obeys the selected distribution law.

The dispersion of a value is described by the normal Gaussian distribution (2) in the case when a large number of random independent parameters affect this value. Most locally distributed random variables in nature satisfy this requirement and, as a rule, can be considered to be normally distributed in analytical and numerical calculations with a sufficient degree of accuracy.

$$\frac{dp}{dx} = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-x_0)^2}{2\sigma^2}} \tag{2}$$

where x_0 is the mathematical expectation; σ is the standard deviation; and σ^2 is the dispersion.

Since the normal distribution is determined on $(-\infty, \infty)$, practical applications require the limited interval, excluding the regions in which the hit probability is close to zero. There are different forms of the normal distribution law. The classical form (2) easily determines the distribution boundaries by the 'three σ' rule (3). In this case, ≈99.7% of random values is within the boundaries of the interval used.

$$x \in (x_0 - \Delta x; x_0 + \Delta x) \quad \text{or} \quad x \in (x_0 - 3\sigma; x_0 + 3\sigma) \tag{3}$$

where $\sigma = \frac{1}{3}\Delta x \approx 0,33\Delta x$

$$P(x_0 - 3\sigma < x < x_0 + 3\sigma) \approx 0.997$$

The normal distribution law (4) can clearly identify the area $x \in (x_0 - E; x_0 + E)$, which includes 50% of random values. In this case, the distribution boundaries that include >99% of random values are determined as $\Delta x = 4E$

$$\frac{dp}{dx} = \frac{\rho}{E\sqrt{\pi}} e^{\frac{-\rho^2(x-x_0)^2}{E^2}} \tag{4}$$

where $E = \rho\sqrt{2}\sigma$

$$\begin{aligned} &\rho \approx 0.477 = \text{const} \\ &P(x_0 - E < x < x_0 + E) = 0.50 \\ &P(x_0 - 2E < x < x_0 + 2E) \approx 0.82 \\ &P(x_0 - 3E < x < x_0 + 3E) \approx 0.96 \\ &P(x_0 - 4E < x < x_0 + 4E) > 0.99 \end{aligned}$$

For the practical interval $\Delta x = 4E$, the hit probability of which is more than 99%, the standard deviation is determined by (5)

$$\begin{aligned} &x \in (x_0 - \Delta x; x_0 + \Delta x) \quad \text{or} \quad x \in (x_0 - 4E; x_0 + 4E) \\ &\sigma = \frac{1}{4\rho\sqrt{2}}\Delta x \approx 0,37\Delta x \end{aligned} \tag{5}$$

In the absence of the experiment, the authors use a 10% interval $\Delta x = 0.1x_0$ which contains more than 99% of the random values to limit the distribution area. Thus, the use of the normal law for distribution easily and mathematically justifies the selection of the required interval and excludes non-physical values of random variables. Moreover, equations (3) and (5) for determining the standard deviation can be considered to be equivalent from a practical point of view.

Exploding cylinder tests

The exploding cylinder test (Figure 1) is a good illustration of the probabilistic approach.[11–13] The experimental arrangement reduces the influence of inevitable asymmetry in geometric and loading parameters and, thus, provides a uniform radial velocity around the entire circumference. Due to the fact that the strain rate is almost the same for all points of the

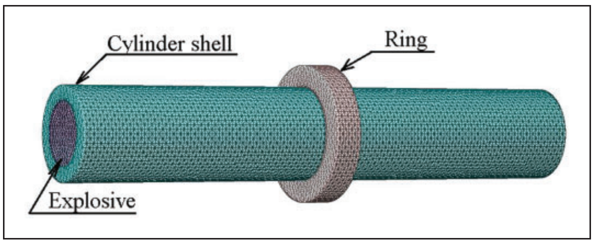

Figure 1. Exploding cylinder test. The experimental arrangement and computational mesh.

ring, its separation into fragments takes place due to the presence of internal inhomogeneities and localisation of strains on them (Figure 2).

A 20 cm long copper tube is filled with explosives. The inner radius of the tube is 1.150 cm, while the outer one is 1.698 cm. Trinitrotoluene was used as an explosive. A ring is put on the middle part of the tube, with an inner diameter equal to the diameter of the tube and an outer radius of 2.5 cm. The height of the ring was 1.0 cm.

The 3D computations were performed using the PS300 finite element software developed at the Research Institute of Applied Mathematics and Mechanics, Tomsk State University, (RIAMM TSU) using the Lagrangian formulation with a tetrahedral mesh.

The system of governing equations solved during the simulation includes the laws of conservation of mass, momentum and energy, the equation of state, relationships, which determine the relations for the deviators of the elastoplastic medium, and the Mises yield criterion.[18] These equations and a computational scheme for their solution are presented in detail in Wilkins.[19] A modified finite element method proposed by Johnson et al.[20,21] was selected as a numerical method for PS300. Automatically generated regular computational mesh (Figure 1) used in these computations contained about 500,000 tetrahedral cells.

To describe fracture, the node splitting method was used; when the fracture criterion (equivalent plastic deformation criterion) was met in the vicinity of the node, node splitting and the formation of the fracture surface take place. The inhomogeneities of the structure were simulated in such a way that the ultimate value of equivalent plastic strain was distributed in the cells of the computational domain by the normal distribution law (4) with dispersion (5) in the range of 10% deviation $x \in (\varepsilon^{eq}_{max} - 0.1\varepsilon^{eq}_{max}; \varepsilon^{eq}_{max} + 0.1\varepsilon^{eq}_{max})$.

The detonator is triggered at one of the ends of the tube and the detonation wave starts propagating along the axis of the copper shell. At the moment when the explosion wave reaches the middle of the shell where the ring is located, the nature of the process begins to change. The ring plays the role of an additional stiffener and inhibits the expansion of the shell exposed to the pressure of detonation products. At the same time, the expansion of the ring takes place. The probabilistic nature is observed at all stages of the ring fracture: first microcracks are formed in the largest structural defects; however, not all of them turn into macrocracks; this requires an appropriate concentration of defects around. Figure 3 shows that the final fragmentation spectrum is in qualitative and quantitative agreement with the experimental results[12] both in the number of formed fragments and in their masses. The graph shows a characteristic break of the curve between large fragments formed by radial cracks and smaller ones formed by the interaction of cracks.

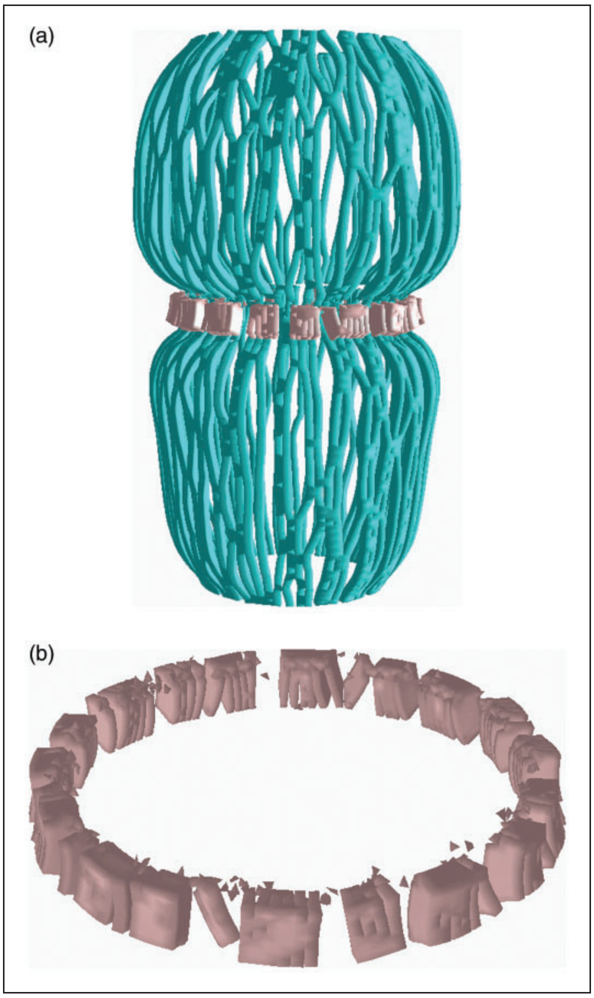

Figure 2. Exploding cylinder test. Numerical results: (a) tube expansion (70 μs) and (b) fragments of the destroyed ring.

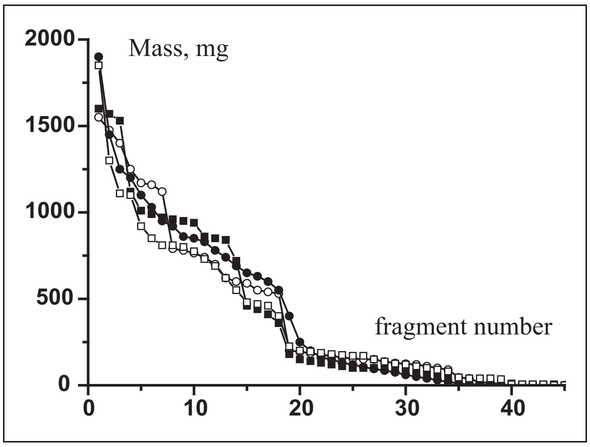

Figure 3. Mass of the ring fragment versus the fragment number; ○, □ – experiment,[12] ● – computation using the basic cell size (Figure 1), ■ – computation using the cell size reduced by 1.5 times relative to the basic one.

The probabilistic nature of fracture during penetration of thin targets by a projectile

Penetration of thin targets from ductile materials, as a rule, is accompanied by petalisation, in spite of the initial symmetry of the task, the edges of the hole turn out to be torn and have the shape of bent 'petals'.[22] The number of 'petals' depends on the impact velocity and the strength material characteristics. Despite uniform and symmetrical loading, in reality, localisation of strains on structural inhomogeneities begins very quickly. In this case, an unloading zone that depends on the strain rate and determines the characteristic size of the 'petal' is formed near growing cracks.

Numerical simulation of the penetration of a thin metal disk by an ogive shape projectile was carried out using the PS300 software. A disk with a diameter of 26.4 cm and a thickness of 2.3 mm is rigidly fixed at the edges. The ogive shape projectile with a diameter of 6.6 cm was considered to be absolutely rigid, which, however, does not play a special role, since the target is too thin and the massive projectile is almost not deformed. The maximum strain criterion was used to describe the fracture. In the framework of the described probabilistic approach, it was believed that each cell had an independent value of ultimate plastic deformation and yield strength. To consider the fracture process to be probabilistic, initial physical and mechanical characteristics responsible for the strength, ultimate strains and yield strength were distributed in the target. The distribution interval (5) of both parameters was limited by a 10% error of the table values.

The computations (Figure 4) clearly show how the number of 'petals' changes with a change in the experimental parameters. For a copper disk: yield strength is 200 MPa and elongation at fracture is 0.563. For a steel disk: yield strength is 940 MPa, elongation at fracture is 0.2.

Due to less ductility, the steel disk (Figure 4(a)) has less time for localisation of fracture in initial structural defects and for unloading when a crack occurs, therefore, at the same projectile velocity 300 m/s it has a larger number of 'petals' (six) than a copper disk (five) (Figure 4(b)). For a copper disk, the computations were performed for different projectile velocities. When the projectile velocity reduces to 150 m/s (Figure 4(c)), the time for fracture localisation and unloading in the case of a crack formation increases, the unloading zone (and, accordingly, the fragment size) increases, so the number of petals decreases to four.

These results show that the proposed probabilistic approach can even in such an axisymmetric problem describe the localisation of strains in structural defects and provide petalisation observed in experiments. The dependence of the number of 'petals' on the projectile velocity and the plastic properties of the target is confirmed.

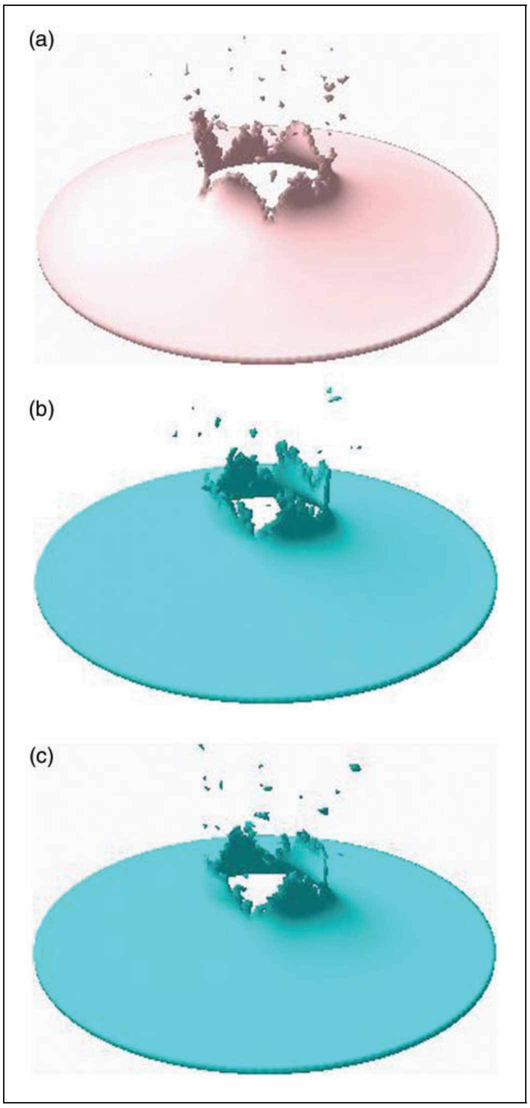

Figure 4. Penetration of a thin disk by a rigid ogive shape projectile: (a) a steel disk, the projectile velocity is 300 m/s, 6 'petals'; (b) a copper disk, the projectile velocity is 300 m/s, 5 'petals'; (c) a copper disk, the projectile velocity is 150 m/s, 4 'petals'.

In order to verify the approach used, the experimental data were compared with the data of numerical simulation.

Figure 5(a) shows a thin metal target penetration carried out using a light-gas gun[23] at the RIAMM TSU. The 1.7 mm thick plate is made of AMg6 alloy. An aluminium spherical projectile with a diameter of 9 mm was accelerated to a velocity of 800 m/s. The experiment shows characteristic torn edges and a rather large (compared to the size of the projectile) hole size due to the deformation of the aluminium ball during penetration.

Using the probabilistic approach shows good agreement between the experimental data and the results of numerical simulation both in the number of 'petals' and in their shape.

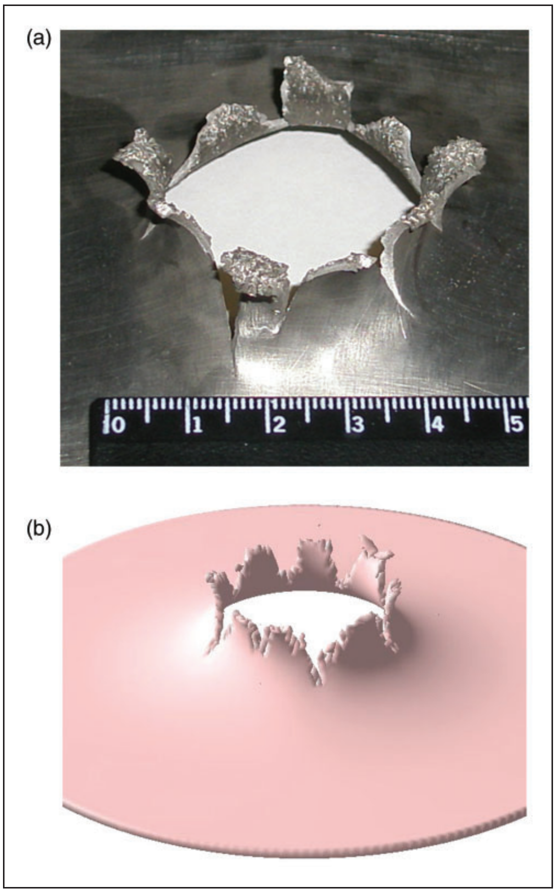

Figure 5. Penetration of a 1.7 mm thick plate from AMg6 alloy by an aluminum spherical projectile with a diameter of 9 mm; an impact velocity is about 800 m/s; (a) experiment; (b) numerical simulation.

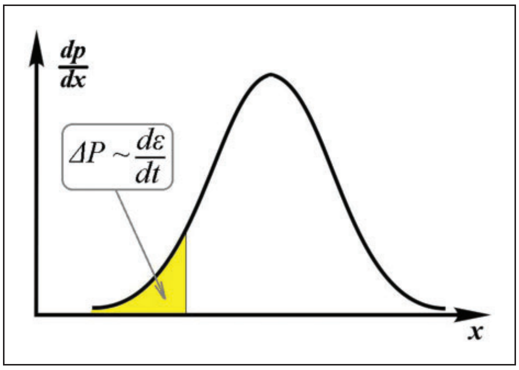

Figure 6. Section of the distribution function of ultimate values that determines the size of the fragments is limited and depends on the strain rate.

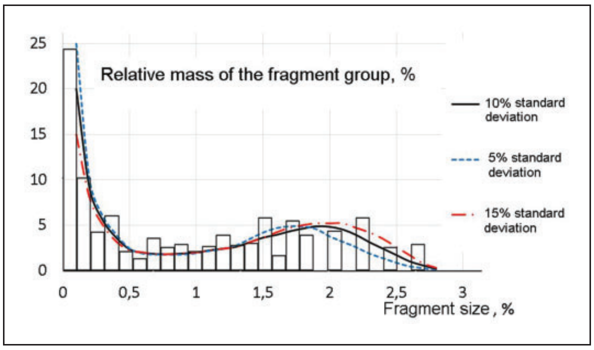

Figure 7. Averaged spectra of fragmentation sizes with different dispersion and typical spectra (presented as histogram).

The effect of the distribution law of strength properties on the fragmentation spectrum (mass distribution)

The selection of the distribution law for the probabilistic approach (normal, exponential, Weibull distribution and other multiparameter distributions) requires additional research. Numerical simulation of the fracture of thick-walled cylindrical shells showed that the dispersion of the initial distribution of strength properties had a stronger effect on the formation of the fragmentation spectrum than the form of the distribution law, which allows any unimodal law to be used for analytical and numerical calculations in the framework of the probabilistic approach.[12]

The spectrum of fragments in each case is determined by the limited region (Figure 6) of the distribution function. The size of this region depends on the strain rate and is determined by the conditions of the propagation of primary unloading waves in the sample. The next stage is the localisation of deformations, and the growth and coalescence of cracks. After that, the shape of the distribution function does not affect the fragmentation spectrum. In fact, the main role of initial inhomogeneities in high-velocity deformation is the stress concentration during the formation of first microcracks, and the strain rate has the main effect on the characteristic fragment size.

Numerical computations were conducted to evaluate the effect of the strength distribution law. The fracture of copper cylindrical thick-wall shells filled with RDX explosive was considered. The diameter of the shell was 6 cm, the diameter of explosive (equal to the inner diameter of the shell) was 4 cm. The critical level of the equivalent plastic strain was chosen as the fracture criterion. The critical values of equivalent plastic strains were distributed to provide a probabilistic approach to the initial distribution of structural inhomogeneities and strength properties of the copper cylindrical shells.

The effect of dispersion was evaluated by the normal distribution law with 5–15% standard deviation ($3\sigma = 5\%\ x_0$; $3\sigma = 10\%\ x_0$; $3\sigma = 15\%\ x_0$, where x_0 is the reference value).

Averaged spectra of fragmentation sizes (Figure 7) correspond to the experimental bimodal distribution. The computations demonstrate an increase in the number of large fragments and a decrease in the number of small fragments with increasing dispersion. Such a feature of shell fracture is due to the fact that with an increase in dispersion, the number of large defects increases and, accordingly, the number of small defects decreases.

To evaluate the effect of the form of the distribution law, the fragment distribution by different laws (normal distribution, exponential distribution, Weibull distribution) was compared. The reference value x_0 and the $3\sigma = 10\%$ standard deviation were used for all distribution laws.

The two-parameter Weibull distribution (1) is called the Mott law for the parameter $\lambda = 1/2$, the exponential law for $\lambda = 1$, the Rayleigh law for $\lambda = 2$, and well approximates the normal distribution for $\lambda = 3.57$ (when the arithmetic mean and the median are equal). The Weibull distribution is not symmetrical; therefore, its representation in an analytical form through mathematical expectation and dispersion is not always possible, which causes certain inconvenience in the computations. The Weibull distribution was used for the parameter $\lambda=2$ (Rayleigh distribution), which in variables x_0 and σ gives (6)

$$\frac{dp}{dx} = \frac{2(x - x_0 + c_2)}{\sigma^2 c_1} e^{\frac{-(x - x_0 + c_2)^2}{\sigma^2 c_1}} \quad \text{where } (x_0 - c_2 \leqslant x \leqslant \infty)$$
$$c_1 = \frac{4}{4 - \pi} \qquad c_2 = \frac{\sigma}{2}\sqrt{\pi c_1} \tag{6}$$

The exponential distribution (7), due to asymmetry, is mainly used for distributions in which time is a parameter. Using it for this comparative analysis is not reasonable, but increasing x_0 in such a way that the arithmetic mean matches the reference value and applying it to the initial distribution of strength properties also leads to the fragment distribution, which qualitatively coincides with the distribution obtained by the normal law.

$$\frac{dp}{dx} = \frac{\sqrt{2}}{\sigma} e^{\frac{-\sqrt{2}(x - x_0)}{\sigma}} \quad \text{where} \quad (x_0 \leqslant x \leqslant \infty) \tag{7}$$

Figure 8 demonstrates the averaged spectra of fragmentation sizes for normal, exponential and Rayleigh distributions. The computations show that in terms of the standard deviation metric, the difference between the three averaged spectra is less than between the computation and its averaging.

The difference in averaging the mass of the shell by the normal and Weibull laws is given by

$$\sigma^*_{weibull} = \sqrt{\sum_{i=1}^{n} \frac{(\overline{f_i^{weibull}} - \overline{f_i^{normal}})^2}{n}} = 1.5\%$$

The difference in averaging the mass of the shell by the normal and exponential laws is given by

$$\sigma^*_{exp} = \sqrt{\sum_{i=1}^{n} \frac{\left(\overline{f_i^{exp}} - \overline{f_i^{normal}}\right)^2}{n}} = 1.8\%.$$

The difference between the computation and shell mass averaging is given by

$$\overline{\sigma^*_{normal}} = \frac{1}{m}\sum_{j=1}^{m}\left(\sqrt{\sum_{i=1}^{n} \frac{\left(f_i^{j\_normal} - \overline{f_i^{normal}}\right)^2}{n}}\right) = 2.3\%.$$

Here, n=30 is the number of intervals in the fragmentation spectrum; m=10 is the number of computations for averaging; $\overline{f_i} = \frac{1}{m}\sum_{j=1}^{m} f_i^j$ is the value in the ith interval of the averaged spectrum function; f_i^j is the value of the spectrum function in the ith interval obtained in the jth computation.

The comparison of the effects of different distribution laws demonstrates that distribution dispersion has a greater effect than the choice of a distribution law.

At the same time, for some problems, an increase in dispersion to a certain value stops affecting the fragmentation spectrum, this is due to the fact that at high strain rates, fracture localisation takes place only with a certain number of microcracks.

Use of this approach may raise the question of the reliability of linking structural defects with the

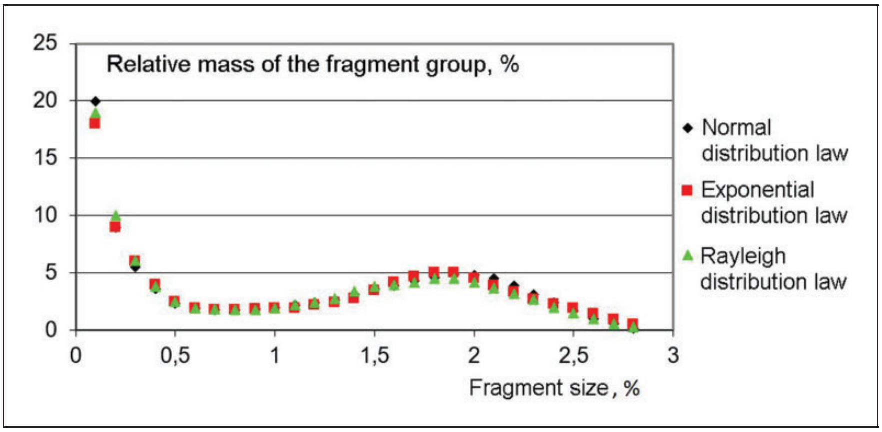

Figure 8. Averaged spectra of fragmentation sizes for three distribution laws and 10% standard deviation.

computational mesh, of evaluating the influence of the size of the computational cell on the fragmentation spectrum, since with a change in the cell size the spatial distribution of initial inhomogeneities will change significantly. This question has been analysed. The selected mesh step, in fact, determines the level of the modeled structure, at the same time, at any level there are distributions of real inhomogeneities that initiate the formation of microcracks. At the atomistic level, dislocations and other defects of the crystal lattice are inhomogeneities. At the mesoscale, grains and grain boundaries are defects. In macroanalysis, microcracks and other large stress concentrators are already inhomogeneities. The distribution of defects for each level can vary. The effect of the size of computational cells at the macro level is shown in Figure 3. It can be seen that with a change (decrease) in the mesh step, the number of large fragments and the characteristic form of the distribution remained unchanged. Thus, the approach with the distribution of strength characteristics is applicable for any cell size; however, as in any other algorithm using computational cells, the cell size must be sufficient to ensure the necessary accuracy.

Conclusion

In this paper a probabilistic approach is proposed for modelling the effect of structural inhomogeneities of a material on dynamic fracture processes in the framework of a phenomenological description of the medium. It is proposed to model internal structural defects and inhomogeneities using the spatial distribution of strength characteristics according to the normal distribution law. The probabilistic approach to modelling dynamic fracture is provided by the introduction of one additional parameter – the dispersion in the distribution of strength material properties. This approach provides the probabilistic nature to the initiation and development of cracks in the material at any scale level: macro, meso and micro level. There are no restrictions on the size of computational mesh and elements, the fracture criterion and the material models to apply this approach. The probabilistic approach is also applicable to multilevel modelling using the appropriate distribution of inhomogeneities. This approach does not require a detailed study of the material structure, which enhances the predictive nature of computations. The numerical results of exploding cylinder tests and the penetration of thin targets by a projectile are presented. Numerical results are in good qualitative and quantitative agreement with experimental data.

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